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Editor

Hello everyone and welcome to another edition of our community's magazine! Hope you all had a good summer vacation and are already full force in the new working year.

I must start by thanking to all the contributors to this issue, and to the community as a whole: in the last issue, I wrote in the editorial note that contributions have been on the low end, but this time I received EIGHT great articles to compose this issue. And we have a bit of everything! Davide Cazzaro tells us about the advances in windfarm optimization, while Christoph Leeters and Anders Reenberg Andersen explain how they model queuing systems where the relocation of customers is allowed. We have our regular contributor Jakob Krarup teaching us about primes and squares, and we have Leonidas Sakalauskas (who is the president of the Lithuanian OR Society and who also contributed in the last issue in the long collection of articles about the war in Ukraine) with a fascinating article about the OR behind social-behavioural and creative societies. We have not one but two Master thesis prize winning articles: Marine Gautier who wont he DORS Prize 2021 and Emil Lindh and Kim Olsson who won the best OR Master Thesis in Sweden for the year of 2021 as well. Last but not least, we have two event reports: the Stockholm Optimization Days 2022 and the Operations and Supply Chain Day 2022 in Aarhus. So, plenty of flavors for all our readers. Unfortunately I could not make it to this years EURO conference in Finland, but I heard from friends who attended that it was quite good! However, what I wanted to stress here is that, given that 2023 is an IFORS year, we can now say that the next EURO-k conference will be in (hopefully sunny) Copenhagen! I know it is still close to 2 years away, but hope to see you all there. Until then, happy reading!

Best, João Fonseca (Editor)

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Svenska Operationsanalysföreningen

Hösten har kommit igen och det är återigen dags att förbereda sig på mer nederbörd, vind och kallare temperaturer. Efter många år i skola och på universitet har jag vant mig vid att hösten är en tid för nystart, när arbete och studier drar igår igen efter sommarens uppehåll. I år känns det också som att vi har fått lite av en nystart efter att pandemin nu har lugnat ned sig, iallafall tillfälligt. Det är givetvis för tidigt att ropa hej och påstå att pandemin är över, men för de flesta i vårt närområde har nu vardagen återkommit efter flera år av hemmasittande, maskbärande och oroande.
Vardagens inträde och normaliseringen av våra arbetssituationer, iallafall till ett 'nytt normaltillstånd', har också märkts i form av att fler och fler konferenser och sammankomster återigen kan hållas i fysisk form. Vissa är fortfarande kombinerade virtuella och fysiska, men flera konferenser under sommaren har varit fysiska, och jag tycker personligen att det har varit väldigt trevligt och upplyftande att återigen kunna träffa kollegor och vänner för att utbyta idéer och tankar.
Ett viktigt lokalt bidrag till konferensfloran i Sverige i år var Stockholm Optimization Days, som anordnades vid KTH i juni. Stockholm Optimization Days innehöll en trevlig blandning av föredrag från de flesta stora akademiska forskningsgrupper och företag inom optimering i Sverige. Ett stort tack till Jan Kronqvist och hans kollegor vid KTH för ett bra och trevligt arrangemang! Snart är det också dags för nästa svenska OR-konferens när Svenska OperationsAnalysKonferensen SOAK återkommer den 24-25 oktober i Stockholm.
Under året har SOAF fortsatt sin seminarieserie med ett virtuellt seminarium per kvartal. Sedan i våras har vi fått lyssna till Kamran Forghani från Uppsala Universitet som talade om optimering av sågning inom skogsindustrin, och senast Edvin Åblad som berättade om sin forskning inom optimerad koordinering och planering av industrirobotar. Ett stort tack till Kamran och Edvin för deras bidrag till vår seminarieserie!

Mattias Grönkvist, Ordförande, SOAF

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Approaches to the wind farm unified optimization

by Davide Cazzaro

The Wind Farm Layout Optimization and the Wind Farm Cable Routing are two of the most important optimization problems in offshore wind farms. In the first problem, we decide the location of wind turbines to maximize their power production. In the second problem, we connect these turbines with electrical cables minimizing their costs. In the literature, the two problems have been studied separately. In the industry, the common approach is to solve the layout problem first and the cable routing second, in a sequential approach. What if we combine the two problems together? After all, the solution to the first problem greatly constraints the solution space of the second one. In this article, we provide an overview of the research approaches to the unified wind farm optimization, and indicate future research lines on the problem.

Figure 1: Horns Rev wind farm in fog conditions, making visible the wake effect. Photo: Vattenfall.

Wind Energy

Wind energy is a renewable source that is clean, sustainable and cost effective. For these reasons, it plays a central role in the European Union strategy to combat climate change and environmental degradation. Since the use and production of energy accounts for 75% of the EU's greenhouse gas emissions, the EU set ambitious goals in the *European Green Deal* to decarbonise the energy system, in order to reach the 2030 climate objectives and achieve carbon neutrality by 2050.

For offshore wind, the EU targets to install 60 GW of offshore wind by 2030, and 300 GW by 2050. Since, as of august 2022, Europe's offshore installed capacity is 28.4 GW [1], substantial development and investments are needed in offshore wind to reach these targets.

During my Ph.D. we focused on optimizing wind farms to extract more power from the wind and to reduce the costs of electrical infrastructure. In particular, we focused on two areas in which mathematical optimization can help wind farms: the first regarding the placement of turbines and the second on the network of electrical cables that connect these turbines.

The two problems of deciding the location of the turbines and connecting them with electrical cables have usually been studied separately. In industry, they are usually solved in sequence, positioning the turbines first and then connecting them with cables. In this article, we will present these two problems and discuss a new direction for the research on wind farm optimization: combining the two problems in a single optimization to take advantage of their mutual influence.

Wind Farm Layout Optimization

The first problem is called the *Wind Farm Layout Optimization* problem. In this problem, we decide the position of the n turbines that we place in the wind farm area. The objective is to find the overall configuration for the turbines that maximizes the power production. Due to the *wake* effect, an aerodynamic phenomenon in which a cone of lower wind speed is generated past the rotor of a turbine, a turbine placed in the wake cone of another turbine will receive slower wind and thus generate less energy, as shown in Figure 1. By accounting for the wind statics on the wind farm site, we aim to find the best placements for the turbines to minimize the wakes and thus maximize the power production. In addition, the Wind Farm Layout Optimization also minimizes the cost of turbine foundations, which depend on the depth and conditions of the seabed. Moreover, we need to respect a minimum distance constraint between turbines, to ensure not only that turbine blades do not collide with each other, but also to maintain the turbulence intensity levels generated in the wake of turbines below an operating threshold. In practice, the minimum distance is often

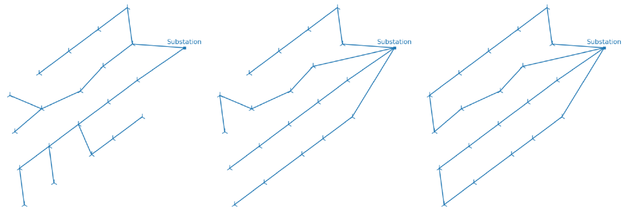

Figure 2: Types of networks for the cable routing problem: branch-based, string-based, and loop-based.

approximated by requiring the turbines to be spaced apart by at least 5 rotor diameters.

By optimizing the position of turbines, we increase the power production of a wind farm using the same number of wind turbines. Indeed, old wind parks placed the turbines in an array configuration, thus generating a high amount of wakes on all the turbines in the center of the park. Due to the application of optimization techniques to this problem, newer parks have turbines scattered in the area in non-geometrical patterns that proved to increase the total power production.

Wind Farm Cable Routing

The second problem is the Wind Farm Cable Routing problem. Turbines are connected with electrical cables placed on the seabed to collect the produced electricity. The cables between turbines are called inter-array cables, and transport the electricity to a central hub, called substation, which is also located offshore. The substation is then connected with an export cable to shore and eventually to the electricity grid. In the cable routing problem, we are given a set of electrical cables with different costs and capacities, and the task is to design the network of inter-array cables with the minimum cost.

Different network configurations can be used: a string-based network, in which at most one cable can enter and exit each turbine, a loop-based network, which connects turbines in rings to ensure a more robust –but also more expensive– solution in case of cable failures, and a branch-based network, in which multiple cables can enter each turbine. We show an example of these networks in Figure 2.

Combined optimization

Instead of solving these two problems in steps, a novel line of research in wind farm optimization is to unify these two problems in a single optimization to reach better wind farm solutions.

We can have an intuition on why it can be desirable to join the two problems by imagining a wind farm area over a large extension of the sea. When solving the first problem, the turbines will spread out as much as possible to minimize the wake effect and thus maximize the power production. However, connecting these far away turbines will then require particularly long cables when solving the second problem. Instead of solving the two problems in sequence, as it was mostly done in the literature and in the industry, we could improve the wind farm solutions by solving a unified problem.

Objective function

In the studies on the problem, the optimization is done as a single objective optimization. In this case, the Net Present Value (NPV) metric is a popular choice.

The NPV is defined as the cumulative discounted cash flows accrued by the wind project over its lifetime:

$$\mathrm{NPV} = \sum_{t=0}^{T} \frac{\mathrm{Revenues}_t - \mathrm{Costs}_t}{(1+r)^t}$$

where T is the wind farm lifetime in years and r is the discount rate. The additive property of the NPV makes it particularly suited to be used as the objective function for the unified problem, because the improvement gap for different solutions of the same park does not depend on how many cost factors are included (which otherwise would be when using other metrics such as the Levelized Cost of Energy). In this way, we can avoid estimating all other costs associated with the park, and include in the objective function only the factors that the optimization does influence, namely the revenues from power production and the costs of turbine foundations and electrical cables.

A new line of research

The unified problem is quite new, and so far three works in the literature have addressed it, to the best of our knowledge. It is worth remarking that both separate problems are NP-hard, making it quite challenging to solve their combination.

A first publication [2], in April 2022, names the problem a *simultaneous design*. In this work, the authors modify a gradient-free heuristic for the layout optimization. Using a fast heuristic, they add a cost estimation of the cable routing associated to the current turbine positions. In this way, the turbine positions take into account the cost of the cables. The cable routing is then refined using a MILP model in a second step.

A second publication [3], in August 2022, calls the problem an *integrated design* and uses a MILP formulation of the two problems. The authors show that it is possible to solve the unified problem, proposing new classes of Benders-like cuts derived from an induced-clique substructure of the problem. They also present

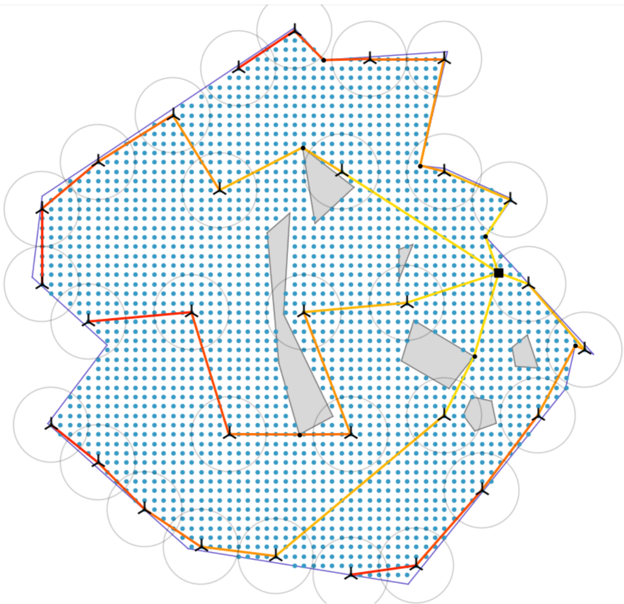

Figure 3: Solution using the sequential approach.

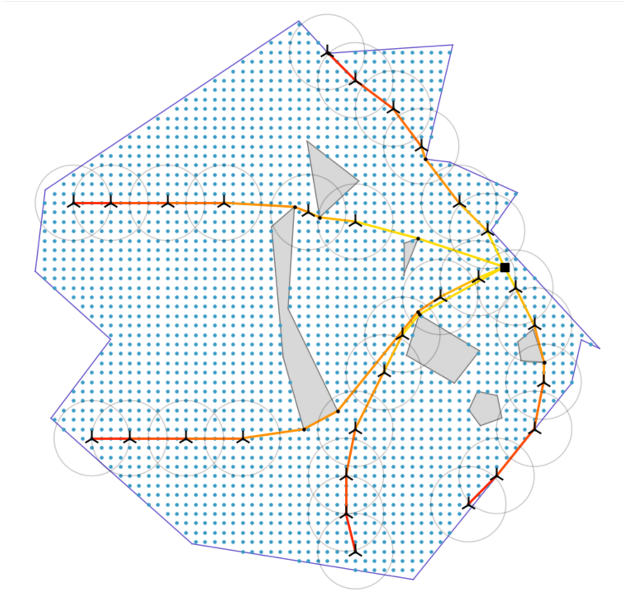

Figure 4: Solution using the combined approach.

an exact branch-and-cut solver to include the cuts effectively at run time.

Finally, in our paper [4] -currently under review- we decided on a heuristic approach, since both the problems are NP-hard. We propose a Large Neighborhood Search that uses a novel combined local search, that modifies both the position of the turbines and the cable network at the same time. In particular, the current solution is modified by moving a turbine to a nearby position while maintaining the current cables connected to this turbine. The destroy and repair phase of the heuristic modifies both the turbines and the cable routing, which is periodically recomputed from scratch.

Both our paper [4] and the gradient-free heuristic paper [2] compare the combined optimization to the sequential approach, reporting that the unified approach does improve wind farm solutions. In particular, by combining the two problems the optimizer balances the power produced by the wind farm and the cost of electrical cables.

We also highlight how the unified optimization is particularly beneficial when obstacles are present in the park. Obstacles are smaller areas that cables cannot cross, due to existing infrastructures or seabed conditions. By solving the two problems together, the additional cable length required to go around obstacles is taken into account when placing the turbines, thus leading to cheaper solutions.

We show a solution obtained with the sequential approach in Figure 3, and a solution for the same wind farm with the combined optimization in Figure 4. We can see that the turbines in the combined approach are not as spread apart as in the sequential solution, and that the positions of turbines lead to a much shorter cable routing around obstacles, thus reaching a higher NPV.

An interesting idea is to formulate the unified problem as a multi-objective problem. It would be interesting, for example, to generate a Pareto frontier to see the trade-off between revenues and costs when optimizing a wind farm. This could help decision makers to take a more informed decision regarding the long term uncertainties of wind farm revenues.

Conclusion

This latest line of research on wind farm optimization looks to be quite promising. Thanks to the collaboration of the Technical University of Denmark and Vattenfall, partners of my Ph.D. project, the optimization tool developed for the unified problem is now used to decide the turbine positions and the electrical network for future wind farms. On real wind farms, the unified optimization proved to save tens of millions of Euros compared to the sequential approach. In this way, the research on wind farm optimization has given a small but concrete contribution to producing cheaper electricity and increasing the wind energy in the European Union.

More research is needed to investigate the best way to combine the two optimizations. A promising idea is to use matheuristic methods that combine heuristics and MIP techniques to solve the unified optimization. Another interesting approach is to consider a multi-objective optimization, which can help highlight the trade-off between revenues and costs in the design of a wind farm.

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Davide Cazzaro holds an Industrial Ph.D. in Operations Reasearch from the Technical University of Denmark. Partner of the Industrial Ph.D. is Vattenfall, an energy and offshore developer company. He currently works in Vattenfall, in the Offshore Wind business unit, where he continues to work on wind farm optimization.

ARTICLE

Queueing systems with relocation of customers

by Christoph Leeters and Anders Reenberg Andersen

The concept of customer relocation occurs in various industries. Customers relocate in the rental industry, where vendors can be substituted, and in hospitals, where patients move between wards to level the stress on the hospital's staff. This article highlights the diverse nature of queueing systems with customer relocation and presents two Markov chain-based approaches for evaluating the systems' behavior. We demonstrate the applicability of our models by applying them to three examples from the industry. The results show that validating these systems leads to transparent decision-making for the business owners and their customers.

Introduction

A Queueing System with Relocation (QSR) is a system consisting of multiple queues of finite capacity. Customers arriving to a queue, where all servers are occupied, will either be lost or relocate to an alternative queue in the system. This type of queueing system constitutes all production, transport, and service systems, where customers can be relocated as an alternative to being lost entirely from the system.

QSRs describe a large handful of systems from the rental industry, including the modern sharing platforms such as electric bike and scooter rentals. Consider the classic example of car rental vendors in an airport. Customers will try to rent a car from their preferred vendor. However, if none of the cars in the vendor's inventory are available, then a fraction of the customers will turn to one of the competitors instead (see Figure 1). Thus, the vendors share the demand between them, where the lack of capacity at one vendor results in increased revenue for the other.

Now let us consider the center of a dense modern city and assume the city contains electric scooters that can be rented using an app. Customers searching for transportation will open the app and get an immediate overview of all available scooters in the area. The customers will choose to rent any available scooters in their vicinity. A fraction of the customers might instead choose to walk a bit further, which essentially corresponds to a relocation. The remaining customers will choose to rent their scooter from a competitor, hire a taxi, or use public transportation, corresponding to a loss from the system.

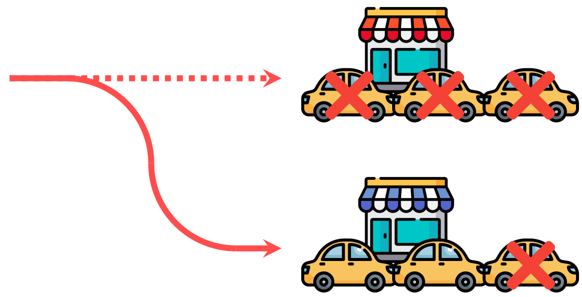

Figure 1: Customer behavior in the car rental example.

QSRs also describe the behavior of inpatient flow in a hospital. Consider a hospital containing a range of inpatient nursing wards. To ensure the best treatment with minimal overcrowding, the hospital has employed a central coordination unit that allocates patients to wards based on the available capacity. On the other hand, the physicians prefer that the patients' diagnoses match the medical specialty of the wards. Thus, the coordination unit prioritizes the preferred ward, but admits the patient to an alternative ward, or a different hospital, if the preferred ward is in shortage of beds. Both scenarios correspond to a relocation of the patient. (see Figure 2).

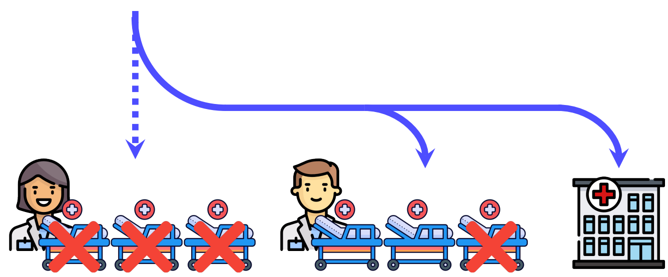

Figure 2: Behavior of inpatient flow in a hospital.

The aforementioned examples reflect some of the most important challenges to modern society. It is, for instance, well-known that sustainable consumption and production are among society's biggest challenges. United Nations stresses that sustainable consumption should be one of the most important goals [1], and the same goes for several scientific studies on the subject [2, 3, 4].

A way to address these problems is to extend the use of sharing economies and other rental systems. In essence, by sharing resources, society benefits from a pooling effect, where fewer resources satisfy the same demand [5]. For this reason, rental services have been investigated for decades. The first study that modeled a rental service as a queueing system was conducted by Tainiter (1964) [6].

Conversely, Benjafaar and Hu (2020) [7] conducted a review of the modern sharing economies and found that many of the concepts are still evolving, leaving a potential for further development of the methodology. A few of the most recent studies include George and Xia (2011) [8] that used a closed queueing network to study the problem of deriving the optimal fleet size in a vehicle sharing platform, and Epstein et al. (2020) [9] that derived the size of a rental inventory with inhomogeneous parameters.

In the healthcare sector, the increasing size and life-expectancy of patients pushes hospitals to derive more efficient ways of maintaining their treatment quality. According to a review of the latest literature by He et al. (2019) [10], the solution is to focus on centralized co-ordination strategies, similar to the example mentioned above. He et al. claim that internal coordination can solve collaboration difficulties and help the individual wards balance their goals. Conversely, the literature on healthcare planning contains very few queueing theoretic studies combining multiple wards. The few examples include Bekker et al. (2017) [11], where a range of bed management strategies are evaluated, and Andersen et al. (2017) [12] where the expected daily number of relocated patients is minimized by optimizing the allocation of beds in a hospital.

QSRs appear in many different ways. The system might depend on a time-inhomogeneous capacity, such as in the electric scooter example, or a large number of queues, such as in the hospital example. In this paper, we restrict our attention to stationary systems with Poisson arrivals, exponentially distributed service times and fixed capacity. In the following section, we present a formal description of the system and our suggestions for an exact and approximate modeling approach. Subsequently, we present three examples of applications, a discussion, and a few concluding remarks.

Exact and approximate models

In this section, we present an exact and approximate approach to modeling the system as a Continuous-Time Markov Chain (CTMC). The CTMC evaluates the state probabilities which in turn can be used to derive essential performance measures of the system, such as the probability of shortage and the expected utilization of the system's capacity.

We start by presenting a formal description of our assumptions. Let $\mathcal{N}$ denote a set of queues and $\mathcal{C}$ a set of customer types. The queues in $\mathcal{N}$ correspond to the facilities, where customers are served (e.g. vendors, inventories or nursing wards). In this paper, we assume that each customer type in the set $\mathcal{C}$ has a unique preference for a queue in $\mathcal{N}$. As a result, we have $|\mathcal{N}| = |\mathcal{C}| = n$. We assume the arrival of customers to the system follows a time-homogeneous Poisson process with rate $\lambda_i \in \mathbb{R}^+$, and that customers will be served with exponentially distributed time with mean $1/\mu_i \in \mathbb{R}^+$, where $i \in \mathcal{C}$.

Let $m_i \in \mathbb{N}$ denote the capacity of queue $i \in \mathcal{N}$, and variable $k_{ij} \in \mathbb{N}_{\geq 0}$ the customers of type $j \in \mathcal{C}$ that are currently served in queue $i \in \mathcal{N}$. Note that k_{ii} corresponds to the number of customers in the preferred queue, and the variable $k_{ij} > 0$, where $i \neq j$, corresponds to customers that have been relocated from queue j to queue i. Queues can only provide service to new customers if they contain idle capacity. That is, if $\sum_{j\in\mathcal{C}} k_{ij} < m_i$. Also, customers of type $j \neq i$ can only be served by queue i if the entire capacity of queue j is occupied. That is, if $\sum_{l\in\mathcal{C}} k_{jl} = m_j$. The relocated customers will choose an alternative queue with probability $p_{ij} \in \mathbb{R}^+$, where $i \in \mathcal{C}$ and $j \in \mathcal{N}$. We assume for convenience that relocated customers are lost if the alternative queue has a shortage of capacity.

Exact formulation

The above formal description of the model yields a system of n queues, where customers are relocated or lost in case their preferred queue's capacity is fully occupied. Let $s \in \mathbb{N}_{\geq 0}^{n\times n}$ denote the current state of the system, and $\mathcal{S}$ its state space. That is, s denotes for every queue $i \in \mathcal{N}$ the number of customers of each type $j \in \mathcal{C}$ that are currently served in queue i. Hence,

$$s = \begin{pmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ k_{21} & k_{22} & \dots & k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{n1} & k_{n2} & \dots & k_{nn} \end{pmatrix}.$$

The occupancies of the queues can be described by a CTMC with the following transition rates:

$$q_{ss^*} = \begin{cases} \lambda_i \text{ if } s^* = (..., k_{ii}+1, ...) \text{ and } \sum_{j\in\mathcal{C}} k_{ij} < m_i \\ \quad \text{in } s. \\ \lambda_j p_{ji} \text{ if } s^* = (..., k_{ij}+1, ...), \text{where} \\ \quad \sum_{l\in\mathcal{C}} k_{jl} = m_j \text{ and } \sum_{l\in\mathcal{C}} k_{il} < m_i \\ \quad \text{in } s. \\ \mu_j k_{ij} \text{ if } s^* = (..., k_{ij}-1, ...) \text{ and } k_{ij} > 0 \\ \quad \text{in } s. \end{cases}$$

Let us examine these transition rates in a bit more detail. The first case $q_{ss^*} = \lambda_i$ describes the arrival rate of a customer of type i at queue i. Therefore, in the new state s^* the number k_{ii} has increased by 1. This state change can only occur if there is still room at queue i for an additional customer, and this condition is described by the constraint $\sum_{j\in\mathcal{C}} k_{ij} < m_i$ in s.

The second transition rate $q_{ss^*} = \lambda_j p_{ji}$ describes a relocation of a type j customer to queue i. Namely, the arrival rate at (the fully occupied) queue j is still λ_j, and queue i is then chosen as an alternative with probability p_{ji}. This relocation clearly leads to an increase of k_{ij} by 1 in the new state s^*. The stated constraints for this relocation describe the obvious conditions that, upon arrival, queue j is fully occupied and queue i is not.

Lastly, the transition rate $q_{ss^*} = \mu_j k_{ij}$ corresponds to a service completion of a type j customer in queue i. This happens with service rate μ_j multiplied with the number of type j customers in queue i, which is k_{ij}. Such a service completion clearly leads to a new state s^* in which k_{ij} has decreased by 1. An obvious condition for this transition is that queue i must contain at least 1 customer of type j, which is denoted by $k_{ij} > 0$ in s.

One can see that the above transition rates are all nonnegative. Furthermore, it holds that the diagonal elements of the transition rate matrix are given by $q_{ss} = -\sum_{s^*\in\mathcal{S}\setminus s} q_{ss^*}$. For all other $s \neq s^*$, we have $q_{ss^*} = 0$, ensuring that all rows of the matrix sum to 0. Hereby the properties of a transition rate matrix are satisfied.

Approximate formulation

A drawback of the above exact formulation of the model is the fact that the state space can become extremely hard to cope with. The size of the state space, $|\mathcal{S}|$, logically depends on the number of queues and customer types, n, and on the queue capacities, m_i. Andersen et al. (2017) [12] showed that the state space size equals

$$|\mathcal{S}| = \prod_{i=1}^{n}\left(\frac{1}{n!}\prod_{j=1}^{n}(m_i+j)\right).$$

This number can become significantly large, already for relatively modest values for n and m_i. To this end, we now present an approximate formulation of the model defined above.

Consider the arrival process of a single queue in the system. As indicated above, customers of the corresponding type arrive at the queue according to a Poisson process. Moreover, arrivals may occur due to relocations of customers of a different type whose preferred queues are fully occupied. This process of relocating customers can be thought of as a Markov-modulated Poisson process. Here, the time between two relocated customers is said to be phase-type distributed. In the remainder of this section, both the Markov-modulated Poisson process and the phase-type distribution will be discussed in more detail.

The phase-type distribution

Consider a CTMC with $p \in \mathbb{N}$ transient states and 1 absorbing state. The transition rate matrix of the CTMC is then of the form

$$\mathbf{Q} = \begin{pmatrix} \mathbf{\Gamma} & \gamma \\ \mathbf{0} & 0 \end{pmatrix},$$

where $\mathbf{\Gamma}$ is a $p \times p$ matrix, and $\boldsymbol{\gamma} = -\mathbf{\Gamma}\cdot\mathbf{1}$ a $n\times 1$ vector denoting the rate at which the absorbing state is reached. Now, the time until the absorbing state of this CTMC is reached, $\mathcal{X}$, is said to be phase-type distributed [13]. This distribution is defined by $\mathcal{X} \sim PH(\boldsymbol{\beta},\mathbf{\Gamma})$. Here, $\boldsymbol{\beta} = (\beta_1, \beta_2, ..., \beta_p)$ is the initial probability distribution of the first p states. The phase-type distribution is a type of matrix-exponential distributions, as can also be seen from its survival function:

$$\mathbb{P}(\mathcal{X} > x) = \boldsymbol{\beta} e^{\mathbf{\Gamma}x}\mathbf{1}.$$

The phase-type distribution is a result of a system of multiple Poisson processes and denotes the total time spent before absorption. In our setting, the time between two relocated customers from queue i to queue j can be thought of as the time until absorption in the CTMC associated with the phase-type distribution.

The Markov-modulated Poisson process

The relocation of customers between queues can be modeled by a Markov-Modulated Poisson Process (MMPP). Here, a Poisson process is controlled by a separate CTMC that is independent of the arrival process [14]. The MMPP is characterized by the transition rate matrix $\mathbf{Q} = \mathbf{C} + \mathbf{\Lambda}$, where $\mathbf{C}$ is a subintensity matrix and $\mathbf{\Lambda}$ the diagonal matrix of arrival rates.

Considering now queue i in our setting. The Poisson process corresponding to arrivals of relocated customers from queue j is interrupted. Namely, these relocations may only occur in case queue j is fully occupied. This is referred to as the MMPP being 'open'. In this case the arrival rate of relocating type j customers at queue i is $\lambda_j p_{ji}$. On the other hand, if there is still idle capacity at queue j, customers of type j will not be relocated and the MMPP is said to be 'closed'. In this case the arrival rate due to relocations is 0.

It would be interesting to know more about the duration of the times during which the MMPP is either open or closed. In the open state, relocations from queue j are only possible if the queue is fully occupied, so the MMPP moves to the closed state as soon as a service is completed. This could be a service completion of any type of customer present in queue j. Now, queue j can be occupied by $\binom{m_j+n-1}{n-1}$ different combinations of customers. Therefore, the MMPP is open for a hyper-exponentially distributed amount of time, where the number of phases equals $\binom{m_j+n-1}{n-1}$ and the absorption rates equal the sum of the service rates of all m_j customers currently present in queue j.

The MMPP is closed as long as queue j contains less than m_j customers. In this case no relocations are possible since queue j has room for at least one more customer. A closed period is terminated by an arrival of a customer in case queue j is servicing $m_j - 1$ customers at that point, i.e. in case there is room for only one additional customer. Now, the distribution of the duration of a closed period is difficult to establish. If queue j would only be subject to Poisson arrivals, this would yield a hyper-exponential distribution [15]. However, arrivals at queue j can also be caused by relocations from other queues, and these relocations may concern customers of different types. The duration of a closed period is therefore not exactly hyper-exponential, but presumably similar to one when the queue is subject to few relocations.

Final model formulation

Lastly, we will present our final approximate model formulation. For this model, we return to the general phase-type distribution. That is, we let go of the theoretically distributed open and closed times of the MMPP. Instead we consider the general phase-type representation $PH(\boldsymbol{\beta}, \boldsymbol{\Gamma})$ as introduced in the beginning of this section. Let queue i denote the queue in 'focus', and let all other queues denote the 'alternatives'. The relocation process of all $n-1$ alternative queues can be modeled by correspondingly $2(n-1)$ phase-type distributions. Note that each alternative queue has one distribution for the open periods and one for the closed periods. The distributions are then chosen to approximate the exact distributions of the MMPP.

The transition rates now become somewhat more involved due to the addition of the phase-type distributions. Denote by γ_{jl}^{open} and γ_{jl}^{closed} the absorption rates of phase l and alternative queue j in the open and closed state, respectively. Similarly, let Γ_{jlr}^{open} and Γ_{jlr}^{closed} denote the rate with which queue j change from phase l to r in the open and closed state, respectively. Also, let β_{jl}^{open} and β_{jl}^{closed} denote the probabilities of queue j entering phase l in the open and closed state, respectively. We further let $t = (k_1, \ldots, k_x, \ldots, k_n)$ replace s, since we only have to track the number of patients in the queue in focus. Moreover, we let $z_j \in \{open, closed\}$ denote the state and $h_j \in \mathbb{N}$ the phase of the alternative queue j. The transition rates, observed from the perspective of the queue in focus, can now be described as follows:

$$q_{ss^*} = \begin{cases} \lambda_i \text{ if } t^* = (..., k_i + 1, ...) \text{ and } \sum_{x \in \mathcal{C}} k_x < m_i \\ \qquad \text{in } t. \\ \lambda_j p_{ji} \text{ if } t^* = (..., k_j + 1, ...), \text{where} \\ \qquad z_j = open \text{ and } \sum_{x \in \mathcal{C}} k_x < m_i \\ \qquad \text{in } t. \\ \mu_x k_x \text{ if } t^* = (..., k_x - 1, ...) \text{ and } k_x > 0 \\ \qquad \text{in } t. \\ \gamma_{jl}^{open} \beta_{jr}^{closed} \text{ if } z_j = closed \text{ and } h_j = r \text{ in } t^*, \\ \qquad \text{where } z_j = open \text{ and } h_j = l \text{ in } t. \\ \gamma_{jl}^{closed} \beta_{jr}^{open} \text{ if } z_j = open \text{ and } h_j = r \text{ in } t^*, \\ \qquad \text{where } z_j = closed \text{ and } h_j = l \text{ in } t. \\ \Gamma_{jlr}^{open} \text{if } h_j = r \text{ in } t^* \text{ and } h_j = l \text{ in } t. \\ \Gamma_{jlr}^{closed} \text{if } h_j = r \text{ in } t^* \text{ and } h_j = l \text{ in } t. \end{cases}$$

The first three transition rates are similar to the initial model. These rates describe the arrivals and departures at queue i. The remaining transition rates reflect the state changes of the alternative queues. The case $q_{ss^*} = \gamma_{jl}^{open} \beta_{jr}^{closed}$ reflects a customer leaving queue j when the queue is fully occupied. The phase of queue j changes and z_j moves from open to closed. As a consequence, relocations from queue j to queue i are no longer possible. The next transition rate $q_{ss^*} = \gamma_{jl}^{closed} \beta_{jr}^{open}$ corresponds to the opposite direction. Queue j moves to an open state and its phase changes accordingly. This means that the arrival process is no longer interrupted and hence customers can be relocated to queue i. The remaining two transition rates describe only phase changes, where neither the number of patients in queue i nor the state z_j of queue j is altered.

Examples of applications

In this section, we present three applications of QSRs to demonstrate the applicability of the exact and approximate model formulations. Our examples of applications cover a case from the construction industry regarding the rental of machinery, a fictional case related to car-sharing, and lastly a case about inpatient flow from a hospital in Denmark.

There exists a small handful of open source tools for evaluating QSRs, and in particular for the type of systems that are covered in this paper. We present an overview below:

- *Bed Evaluation & Design System (BEDS)*. Designed for evaluating inpatient flow using either simulation or a CTMC approximation. An executable file for Windows is available on Sourceforge[1].

[1] BEDS: sourceforge.net/projects/beds

- *RelSys*. Evaluates QSRs assuming time-homogeneous parameters. RelSys employs the above-mentioned approximate CTMC formulation using hyper-exponential phase-type distributions. RelSys is also the algorithm behind BEDS and is written in C++. The source code is available on GitHub[2].
- *TranReloc*. A flexible tool for evaluating QSRs assuming time-inhomogeneous parameters. This feature is particularly useful for systems, where the demand fluctuates over time. TranReloc also gives the user the option to employ state dependent relocation rules. The source code is written in Java and available on GitHub[3].

Rental of machinery

This example is based on an application of queueing theory from the company NetHire A/S. NetHire provides a platform for various vendors to manage and rent their tools, equipment, and machinery to craftspersons and entrepreneurs. In this example, we consider the rental of scissor lifts from one of NetHire's vendors.

The vendor keeps an inventory of scissor lifts from two manufacturers, Dingli and JLG. The lifts are otherwise almost identical – they are both electric and have a maximum height of 7.8 meters. The majority of customers preferring one brand can use the other as an alternative since the lifts substitute each other in terms of function. The vendor wants to minimize the holding cost of the inventory, and is therefore interested in the shortage probability and mean occupancy of both products.

The demand for scissor lifts are $\lambda_{Dingli} = 0.8$ and $\lambda_{JLG} = 0.32$ lifts per day, and the lifts are occupied for an average of $1/\mu_{Dingli} = 6.061$ and $\mu_{JLG} = 4.975$ days, respectively. We assume customers leave the system with a probability of 0.1 if they encounter a shortage of their preferred lift, and the remaining customers relocate to the alternative lift. The vendor's inventory contains $m_{Dingli} = 14$ and $m_{JLG} = 5$ lifts, respectively.

We evaluated the solution to the exact CTMC using the Matlab script from the *Exact evaluation* folder in the RelSys source code. Table 1 shows the shortage probability and mean occupancy of both lifts.

	Scissor lift 7.8 m, Dingli	Scissor lift 7.8 m, JLG
Inventory size	14	5
Shortage prob.	$3.811 \cdot 10^{-4}$	$1.753 \cdot 10^{-2}$
Mean occupancy	4.872	1.565

Table 1: Evaluation of the shortage probability and mean occupancy for lifts of different brands. The system is evaluated using the exact CTMC formulation.

Car-sharing

In this example, we consider a fictional city where the citizens can subscribe to a car-sharing service, which provides them with a flexible method of transportation. The citizens may use the car as long as they need but have to return to the same parking lot. We assume that citizens use an app to get an overview of all available cars and that they immediately reserve their preferred choice.

The city can be divided into a grid containing 2×2 zones, where each zone contains a constant capacity of cars. Figure 3 specifies the distribution of capacity across the four zones. The citizens reserve the cars that are closest to them, and as a result, they will always choose a car from their current zone or one of the neighboring zones. Note that all zones have two equidistant neighbors, and one diagonal neighbor, which is a bit further away. If all cars are occupied in a citizen's current zone, the citizen will choose an alternative method of transportation with a probability of 0.5. Otherwise, the citizen will choose to walk for a car in one of the two equidistant zones (with equal probability if both zones are available). The citizen will only choose the diagonal neighbor if all other options are exhausted.

Zone 0,0 1 car	**Zone 0,1** 5 cars
Zone 1,0 2 cars	**Zone 1,1** 10 cars

Figure 3: The four zones and distribution of cars in the city.

The citizens occupy the cars for an average of $1/\mu = 0.5$ hours, and the demand for cars in each zone is: $\lambda_{0,0} = 2$, $\lambda_{0,1} = 8$, $\lambda_{1,0} = 5$, and $\lambda_{1,1} = 9$ cars per hour.

Note that in this particular case, the relocation probabilities depend on the zones that are fully occupied at the time of arrival. Thus, we must employ the TranReloc tool to evaluate the exact solution to the system. Table 2 contains the resulting shortage probabilities, and compare the solution to a simpler system assuming that citizens relocate with equal probability to one of the two closest zones, and never to

[2]RelSys: github.com/areenberg/RelSys
[3]TranReloc: github.com/areenberg/TranReloc

the diagonal zone. The solution to the simpler system, denoted *fixed*, is evaluated using the above-mentioned exact and approximate CTMC formulations. To evaluate the former, we use the TranReloc tool, and for the latter, we use the BEDS tool.

	City zones			
	0,0	0,1	1,0	1,1
Number of cars	1	5	2	10
Exact (state dep.)	0.573	0.220	0.494	0.020
Exact (fixed)	0.570	0.206	0.486	0.013
Approximation (fixed)	0.570	0.211	0.488	0.016

Table 2: Evaluation of the shortage probability in each of the four city zones. Evaluated with two assumptions (state dependent and fixed relocation) using both the exact and approximate CTMC formulations.

Hospital inpatient flow

In this final example, we demonstrate the applicability of QSRs to the healthcare industry by replicating the study by Andersen et al. (2017) [12]. The study considers three inpatient nursing wards from a Danish hospital covering diseases in four areas – i.e. gastrology, pneumology, endocrinology, and geriatrics. Similar to our example in the introduction, the patients can be relocated to an alternative ward if their preferred ward is fully occupied upon arrival at the hospital. Accounting for the inter-dependence between the wards, Andersen et al. derive the allocation of beds that minimizes the expected number of relocations per day.

Table 3 and 4 contain the arrival rates, mean length-of-stays and relocation probabilities for the patients associated with the three wards.

Pat. type	Preferred ward	Arrivals per day	Length-of-stay in days
1	Ward 1	5.42	5.26
2	Ward 2	3.96	5.26
3	Ward 3	2.52	9.09

Table 3: Arrival rates and mean length-of-stays for the three patient types.

	Ward 1	Ward 2	Ward 3
Pat. type 1	-	0.05	0.23
Pat. type 2	0.10	-	0.27
Pat. type 3	0.06	0.00	-

Table 4: Relocation probabilities for all patient types and wards.

The wards contained $m_1 = 27$, $m_2 = 23$ and $m_3 = 24$ beds prior to the optimization, which in the exact CTMC formulation leads to $\approx 3.09 \cdot 10^{10}$ states. Storing the entire exact solution in double precision would require 247.2 GB of memory. For this reason, Andersen et al. evaluate the solution by employing an algorithm that truncates the state space. In this paper, we compare the approximate CTMC formulation to the truncation by Andersen et al. and validate both methods by simulating the shortage probabilities. We use the BEDS tool for both of these tasks.

The results are presented in Table 5, showing the shortage probabilities for the wards before and after the optimization. The optimized bed allocation reduces the cases of relocation from 1.804 to 1.592 patients per day [12].

	Beds	Andersen et al. (2017)	Approximation	Simulated (95% conf. int.)	Scenario
Ward 1	27	0.178	0.176	0.179 (0.177;0.181)	
Ward 2	23	0.109	0.106	0.109 (0.107;0.110)	*Before opt.*
Ward 3	24	0.161	0.153	0.165 (0.162;0.167)	
Ward 1	32	0.083	0.085	0.084 (0.083;0.085)	
Ward 2	24	0.084	0.084	0.082 (0.081;0.084)	*After opt.*
Ward 3	18	0.318	0.322	0.322 (0.319;0.324)	

Table 5: Evaluation of the shortage probabilities using the truncated CTMC from Andersen et al. (2017) [12], the approximate CTMC, and a simulation of the inpatient flow.

Discussion

The examples addressed in the previous section show that both the exact and approximate formulation of the CTMC are suitable for modeling QSRs. In the machinery rental example, we can see that the shortage probability for both lifts is negligible. Furthermore, the mean occupancy lies well below the current inventory size. These results show that the inventory sizes can be lowered in order to reduce the holding costs. It must be noted though, that the probability of customers leaving the system when their preferred lifts are in shortage is assumed to be 0.1. Clearly, picking a different value would result in different outcomes. However, as we have used the exact CTMC approach here, the results are known to be accurate.

The application of car-sharing, despite having a somewhat simplistic setup, is well suitable for a CTMC modeling approach. The exact formulation with fixed relocations adequately represents the system, and the approximated formulation only deviates slightly. The obtained results show that Zone 1,1 clearly has the lowest shortage probability. This indicates that a more balanced distribution of cars would be preferable, although this may not be a possibility. Nevertheless, customers could benefit from this result by preferring Zone 1,1 to the other relocation zones.

Finally, in the last example, the inpatient flow for a Danish hospital is modeled by the approximate CTMC formulation. We can see that the results from both the inpatient flow simulation and the approach by Andersen et al. are closely approximated by the CTMC model. Due to memory issues it remains difficult to implement the exact CTMC formulation, but the approximate alternative proves to be well suitable. Both before and after the optimization namely, the shortage probabilities of all three wards are accurately estimated by the approximate CTMC approach.

Concluding remarks

We conclude that a CTMC modeling approach is well applicable to QSRs. After formulating both an exact and an approximate CTMC model, we applied these on several practical examples to evaluate in particular the shortage probabilities of the queues. The applications of machinery rental and car-sharing showed that the obtained shortage probabilities can help a vendor or customer make adequate decisions. In the application to hospital inpatient flow we compared our proposed approximated CTMC approach to a simulation and an alternative approximation by Andersen et al. Here, we saw that the shortage probabilities obtained by the alternative methods were closely approximated by our model, indicating that the CTMC approach is well suitable for modeling QSRs.

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An essay on primes and squares

by Jakob Krarup

Subtract 2 from the square of an odd integer greater than 1. Subtract 2 from the square of an even integer 2x and divide the difference by two. As will be shown, the result will in both cases either be a prime or the product of primes of which the smallest is at least 7. Furthermore, for squares up to 101×101, the proportion of primes among $x^2 - 2$ or $2x^2 - 1$ appears to be at least one half. Mathematical prerequisites: nothing beyond what hopefully was learnt at the elementary school.

Motivation

"Hey, look at the car in front of us. The number on its license plate is divisible by 13 and almost a square!" Knowing that she is married to a number freak, my wife is neither impressed nor surprised. Actually, numbers have been a life-long passion. I cannot come across a number, in particular an integer, without watching out for its potential properties, notably its divisors.

A pertinent keyword for this essay is *mental arithmetic*. To identify divisors of a given integer without access to any aids, the significance of being within the neighbourhood of a square is emphasized. To be more specific, the properties of $x^2 - 2$ or $2x^2 - 1$ where x is an odd or even integer, respectively, will be accounted for.

During the early school days, our math teacher taught us simple rules for detecting whether an integer is divisible by the small primes 2, 3, 5, 11; furthermore, these rules were easily extended to encompass 9, and powers of 2 or 10 as well. On the other hand, and to our disappointment, we were also told that no such simple rules exist for 7 and 13. Since $1001 = 7 \times 11 \times 13$, however, this statement is only partially true. This postulate is accounted for in [1] where it amongst several other observations is shown that catching a glimpse on a 5-digit integer, for example with a zero in the middle, is sufficient to decide whether it is divisible by any of 7, 11, 13.

While searching for divisors of a given integer p, it is often helpful to identify squares x^2 where x^2 is greater than or equal to p. The point is that $x^2 - 1 = (x+1)(x-1)$ or, in more general terms, that $(x^2 - y^2) = (x+y)(x-y)$ where y can be any real number.

Let p be an odd integer greater than 1. There are three possibilities, a) p is a prime, b) p is a square, and c) p is the product of two odd integers q, r where $q > r$. In case c), p must equal the difference between two squares. For example, $91 = 13 \cdot 7 = (10+3)(10-3) = 10^2 - 3^2$, where 10 and 3 are determined as $\frac{1}{2}(13+7)$ and $\frac{1}{2}(13-7)$, respectively. For $p = qr$, the general expression reads:

$$[\tfrac{1}{2}(q+r)+\tfrac{1}{2}(q-r)] \times [\tfrac{1}{2}(q+r)-\tfrac{1}{2}(q-r)] = (\tfrac{1}{2}(q+r))^2 - (\tfrac{1}{2}(q-r))^2 = \tfrac{1}{4}(q^2 + r^2 + 2qr - q^2 - r^2 + 2qr) = qr$$

Thanks to our dedicated math teacher, all squares up to 25×25 are still etched into my memory. Thus, for modest values of p, and, unless p is a prime, no pocket calculator is required as it often is fairly straightforward to check whether p is a square or happen to equal the difference between two squares.

Why "... up to 25×25" only? $24^2 = 576$, and $26^2 = 676 = 24^2 + 100$. In general, $(25+y)^2$ can be determined as $(25-y)^2 + 100y, y = 1, 2, ..., 25$. Similarly, $51^2 = 2601 = 50^2 + 1 + 100$, or, in general, $(50+y)^2 = 50^2 + y^2 + 100y$. Thus, the list of "squares to be remembered" extends to 50+.

Assume that the integer p to become investigated is even and not a power of 2. Divide p by 2 and repeat this step until the quotient is odd. Henceforth, and although some observations in the next section apply for all integers, our main aim in the first round is to focus on potential divisors of odd numbers only.

We are now homing in on the target. For p odd, and, unless p itself is a square, let x^2 be the smallest square greater than p. Neither of $x^2-1, x^2-4, x^2-9, x^2-16, ...$ can be primes, cf. $(x^2 - q^2) = (x+q) \times (x-q)$. Furthermore, no primes are found among $x^2 - 3, x^2 - 5, x^2 - 7, ...$ which all are even. A good question is then: what about $x^2 - 2q$, where $q = 1, 2, ...$, and $2q$ is not a square?

On the equation $x^2 - 2 = 3y$

Let x be any integer greater than 1. We shall prove that $x^2 - 2$ is not divisible by 3. To this end, x is rewritten as $x = 3\alpha + \beta$, where α is an integer, and $\beta = 0, 1, 2$. Hence,

$$x^2 = 9\alpha^2+\beta^2+6\alpha\beta \Leftrightarrow x^2-2 = 3(3\alpha^2+2\alpha\beta)+\beta^2-2$$

Thus, 3 divides $x^2 - 2$ if and only if 3 divides $\beta^2 - 2$. For the three values of β we obtain

$$\beta = 0 : \beta^2 - 2 = -2$$

$$\beta = 1 : \beta^2 - 2 = -1$$

$$\beta = 2 : \beta^2 - 2 = 2$$

$\Rightarrow x^2 - 2$ is not divisible by 3. Let the result be stated as

Fact 1. There are no integers x, y for which $x^2-2 = 3y$.

Let x be redefined as any odd integer greater then 1. We will show that $x^2 - 2$ is not divisible by neither 2 nor 5. x odd implies that x^2 is odd; hence, $x^2 - 2$ is not divisible by 2. 5 divides an integer if and only if the last digit is either 0 or 5. 0 can be excluded since x is odd. As regards 5, the last digits of $x^2 - 2$ form the repeated sequence 7-3-7-9-9. Hence, 5 can be excluded as well. We have thus proved,

Fact 2. For any odd integer $x > 1$, $x^2 - 2$ is either a prime or the product of primes of which the smallest is greater than or equal to 7.

On the proportion of primes among $x^2 - 2$, x odd and $3 \leq x \leq 101$

As was expected, several primes can be written as x^2-2 for x odd. To be more specific, it will be shown that

Fact 3. For x odd and $3 \leq x \leq 101$, the proportion of primes among $x^2 - 2$ equals one half.

The proof reduces to presenting Table 1, showing the relevant figures.

x	x^2	x^2-2	**P**	**divisors**	**new**
3	9	7	•		
5	25	23	•		
7	49	47	•		
9	81	79	•		
11	121	119		7, 17	7, 17
13	169	167	•		
15	225	223	•		
17	289	287		7, 41	41
19	361	359	•		
21	441	439	•		
23	529	527		17, 31	31
25	625	623		7, 89	89
27	729	727	•		
29	841	839	•		
31	961	959		7, 137	137
33	1089	1087	•		
35	1225	1223	•		
37	1369	1367	•		
39	1521	1519		7, 7, 31	
41	1681	1679		23, 73	23, 73
43	1849	1847	•		
45	2025	2023		7, 7, 17	
47	2209	2207	•		
49	2401	2399	•		
51	2601	2599		23, 113	113
53	2809	2807		7, 401	401
55	3025	3023	•		
57	3249	3247		17, 191	191
59	3481	3479		7, 7, 71	71
61	3721	3719	•		
63	3969	3967	•		
65	4225	4223		41, 103	103
67	4489	4487		7, 641	641
69	4761	4759	•		
71	5041	5039	•		
73	5329	5327		7, 761	761
75	5625	5623	•		
77	5929	5927	•		
79	6241	6239		17, 367	367
81	6561	6559		7, 937	937
83	6889	6887		71, 97	97
85	7225	7223		31, 233	233
87	7569	7567		7, 23, 47	47
89	7921	7919	•		
91	8281	8279		17, 487	487
93	8649	8647	•		
95	9025	9023		7, 1289	1289
97	9409	9407		23, 409	409
99	9801	9799		41, 239	239
101	10201	10199		7, 31, 47	

Table 1: Proof of Fact 3

Only the last three columns call for an explanation. The primes found are marked by • in column **P**. If $x^2 - 2$ in the third column is not a prime, its divisors are listed in the column labelled by **divisors**. Divisors appearing for the first time are the contents of the rightmost column called **new**. Note that no divisor less than 7 were found, cf. Fact 2.

25 out of the 50 entries considered are primes corresponding to a proportion of 50% or one half whereby we are done.

Recall that neither of $x^2 - q^2, q < x$, can be primes as $(x^2 - q^2) = (x + q)(x - q)$. Furthermore, for q odd and less that x^2, no primes are found among $x^2 - q$ which all are even. Thus, in the search for primes expressible as $x^2 - y$, y being an integer, the only case of interest is $x^2 - 2q$, where $q = 1, 2, \ldots$, and $2q$ is not a square.

The case $q = 1$ or $x^2 - 2$ has already been accounted for. For $q = 2, 2q$ is a square. Now, what about $q = 3$ or $x^2 - 6$? Though not shown here, a table has been compiled for $x^2 - 6$. As might be expected, the number of primes found among $x^2 - 2q$ will decrease for increasing values of q. For $q = 3$, 17 of the 50 entries are primes. This corresponds to a proportion of 34% which is less than the 50% found for $q = 2$.

On the equation $2x^2 - 1 = 3y$

Fact 2 deals with odd integers only. Is it conceivable that a similar result can be obtained for even integers as well? Adopting the same approach as in the previous two sections, this question will now be investigated.

Let x be any positive integer and consider the expression $(2x)^2 - 2 = 2(2x^2 - 1)$ where $2x$ represents an even integer. Certainly, $2(2x^2 - 1)$ is divisible by 2 and cannot be a prime. Now, what about $2x^2 - 1$?

Fact 1 asserts that $x^2 - 2$ is not divisible by 3. Hence, the same property applies for $2x^2 - 4 = 2x^2 - 1 - 3$, and, finally, for $2x^2 - 1$ as well. Thus, there are no integers x, y for which $2x^2 - 1 = 3y$.

Furthermore, $2x^2 - 1$ is not divisible by 2. As 5 is regarded, the last digits of $2x^2 - 1$ form the repeated sequence, 1-7-7-1-9. As both 0 and 5 are absent, $2x^2 - 1$ is not divisible by 5. We have thus proved,

Fact 4. For any integer $x > 1, 2x^2 - 1$ is either a prime or the product of primes of which the smallest is greater than or equal to 7.

On the proportion of primes among $2x^2 - 1, 1 \leq x \leq 50$

Once again, a table is presented to illustrate the situation (Table 2).

Upon exclusion of $p = 1$, the topmost entry in 2, 28 out of the 50 entries considered are primes correspond-

x	x^2	$2x^2 - 1$	**P**	**divisors**	**new**
1	1	1			
2	4	7	•		
3	9	17	•		
4	16	31	•		
5	25	49		7, 7	7
6	36	71	•		
7	49	97	•		
8	64	127	•		
9	81	161		7, 23	23
10	100	199	•		
11	121	241	•		
12	144	287		7, 41	41
13	169	337	•		
14	196	391		17, 23	17
15	225	449	•		
16	256	511		7, 73	73
17	289	577	•		
18	324	647	•		
19	361	721		7, 103	103
20	400	799		17, 47	47
21	441	881	•		
22	484	967	•		
23	529	1057		7, 151	151
24	576	1151	•		
25	625	1249	•		
26	676	1351		7, 193	193
27	729	1457		31, 47	31
28	784	1567	•		
29	841	1681		41, 41	
30	900	1799		7, 257	257
31	961	1921		17, 113	113
32	1024	2047		23, 89	89
33	1089	2177		7, 311	311
34	1156	2311	•		
35	1225	2449		31, 79	
36	1296	2591	•		
37	1369	2737		7, 17, 23	
38	1444	2887	•		
39	1521	3041	•		
40	1600	3199		7, 457	457
41	1681	3361	•		
42	1764	3527	•		
43	1849	3697	•		
44	1936	3871		7, 7, 79	79
45	2025	4049	•		
46	2116	4231	•		
47	2209	4417		7, 631	631
48	2304	4607		17, 271	271
49	2401	4801	•		
50	2500	4999	•		

Table 2: Proof of Fact 5

ing to a proportion of 56%. Note that $p = 7$ is the only prime appearing in two different tables, cf. Tables 1 and 2, respectively. Thus,

Fact 5. For the 50 integers $x, 1 \leq x \leq 50$, the proportion of primes among $2x^2 - 1$ exceeds one half.

The absent primes

Starting from the smallest prime, and in increasing order, the primes less than 100 and equalling one among $x^2 - 2$, $x^2 - 6$, or $2x^2 - 1$ are

$$3, 7, 17, 19, 23, 31, 43, 47, 71, 79, 97$$

as found in the tables above. What about the *absent* primes, say, less than 100:

$$2, 5, 11, 13, 29, 37, 41, 53, 59, 61, 67, 73, 83, 89$$

Consider what may be termed "the inverse problem". Let p be a prime not found among $x^2 - 2$ or $x^2 - 6$. Question: does there exist an integer y for which $x^2 - y = p$?

Fact 6. For any prime p there exists infinitely many pairs of integers x, y for which $x^2 - y = p$.

Proof. For given p, let x be any integer for which $x^2 > p$ and define y as $x^2 - p$. Hence, $p = x^2 - y$.

Figure 1: Euclid, around 300 B.C.: "The number of primes is infinite!"

Acknowledgment

A first draft of the present essay was sent to Professor Kees Roos, Technical University of Delft, The Netherlands, for his comments. My original proof of Fact 1 was correct but rather lengthy. Kees Roos, however, should be credited for having suggested an integer x expressed as $x = 3\alpha + \beta$, as done above; an example of so-called *modular arithmetic*. This reduced the proof to but a few lines of math, still, as Kees pointed out, not beyond what hopefully was learnt at the elementary school. To lend credence to this postulate, recall that elementary school pupils learn to watch the clock.

Literature

Primes have challenged the minds of many for thousands of years and generated a huge literature. There may well have been forerunners, but around 300 B.C., Euclid proved that the number of primes is infinite. The largest known prime (by 2016) is $2^{74.207.281} - 1$, a number with more than 22 millions digits. Approaches spanning from rather simple observations to heavy math have ever since revealed an abundance of other properties applying for primes.

Adequate credit should always be given to those who deserve it. It is difficult, or quite impossible, to believe that nobody ever before has embarked from $x^2 - 2$ or $2x^2 - 1$ in a study of primes. Yet, all search for relevant literature on the internet or via personal communication has so far been of no avail.

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Operational Research Behind Social-Behavioural Phenomena in Creative Societies

by Leonidas Sakalauskas

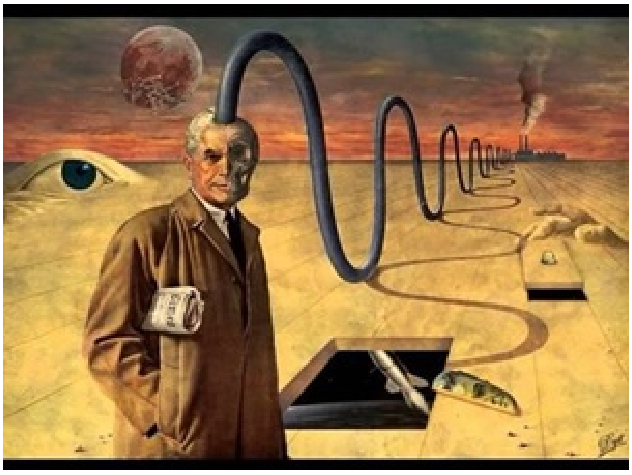
Introduction

Operational Research (OR) is defined as a mathematical or, more broadly, a systematic scientific study on the efficiency of the use of funds, resources, personnel and equipment (Gass and Assad, 2005), which found broad application in many aspect of people's lives. OR was heavily and effectively used throughout the course of the Cold War (Weintraub, 2017), emphasizing the role of game theory and rational decision making in shaping the strategic thinking in politics and business. It is natural to assume that in the current historical time, when many delicate socio-political challenges have to be dealt with once again, it is important to resort to OR to overcome those challenges.

The application of computational models to study issues in the social sciences and humanities has been undergoing rapid development during the last decades. The 1st International & EURO Mini Conference "Modelling & Simulation of Social-Behavioural Phenomena in Creative Societies" (MSBC-2019) has launched the series of international MSBC-20xx conferences aimed at deepening the understanding of OR role in social processes (`http://www.msbc2019.mii.vu.lt`).

In this regard, simple and universally acceptable assumptions behind OR need to be formulated addressing the complex social-behavioural phenomena of today. People operate in accordance with other people's actions and therefore need effective communication based on mutual understanding and rational choice. In parallel to OR, a conception of the operationalisation of physical phenomena in the real world was developed, which later laid the foundations for theory of communicative action (Habermas, 1984) that became a tool for analyzing language (Chomsky, 1956) and action as fundamental elements of the existence of humans, which manifest through communication based on logical and verbal intelligence.

Operationalisation is defined as scientific methodology of operational definition, where even the most basic concepts are validated through their measurement. It originated with the book "The Logic of Modern Physics" (1927), by Percy Williams Bridgman, whose methodological position is called operationalism. Operationalism is based on the assumption of the reality study that the quantitative concepts defining the object of reality can and must be defined by reference to the rules of direct or indirect measurement of that object. P. W. Bridgman insisted that all scientific concepts be described on the basis of measuring procedures, thinking operations and manipulation of symbols. The concept which is not directly or indirectly linked to any measurement procedure is meaningless. Measurement can also be understood in a broader sense: it can be not only the results of monitoring devices or sensors but also the information provided by public surveys or various statistical sources, etc. This approach makes it possible to develop a methodology for researching complex social processes and humanities.



Thus, operationalisation and operationalism are closely linked, but operationalism essentially follows to a philosophy of science, saying that a concept is nothing more than what is observable, and operationalisation is a process for the introduction, monitoring and measurement of quantitative indicators in order to examine theoretical concepts. It should be stressed that operationalisation through mathematical ontologies inevitably leads to limitations relating to the finiteness of this process, the internal organisation of ontologies, the conceivable non-existence of harmonising decisions or solutions to the problems addressed (Arrow, 1950, Kleene, 1951), etc.

Operationalisation in the classic OR was not overemphasised, since the examination of business and engineering systems involves relatively well-defined and clearly understandable technical-economic concepts. Recently, however, there have been a growing number of challenges related to processes that are taking place thanks to interactions between people (their communities). Society creates a set of social behavioural phenomena (e.g., the conduct of voters during elections; the behaviour of the crowd during a terrorist attack; people's behaviour during pandemics; institutions' social capital development processes; the behaviour of social network users; etc.), whose knowledge of mechanisms is essential for building a sustainable society. We aim to review the operationalisation of such phenomena as the expression of logical and verbal communication in order to develop a paradigm of rational choice for the efficient understanding of their nature and relevant decisionmaking, thus bridging OR with social sciences and humanities. The creation of such a paradigm on the basis of the theory of structural equation modelling (SEM), multi-agent modelling and game theory, together with data science and mathematical sociology methods, allows the development of data-driven operationalisation for evidence-based solutions.

Structural equation modelling behind operationalisation in the social sciences and humanities

SEM is a powerful tool for the operationalisation and understanding of complex social processes and humanities. SEM is a methodology for representing, estimating, and testing a theoretical network of (mostly) linear relations between variables (Kaplan, 2008). It can also be used as a decision-making tool, as it includes various sets of mathematical models, computer algorithms and statistical methods to study data constructs. SEM uses a measurement model that defines latent variables related to one or more observed variables and a structural model that presents relationships between latent variables (see Figure 1). The relationship between observable and hidden data and structural equations is assessed by statistical methods. SEM combines regression and factorial analyses.

It should be emphasised that SEM is a good tool for exploring various types of intangible capital such as intellectual, human, social and other capital (Adams and Oleksak, 2010; Sakalauskas et al., 2021). The multidimensional models of emotion recognition give another example of operationalisation in humanities (Russell, 1980; Karbauskaitė et al., 2020). Emotional recognition in images or texts is a common problem of artificial intelligence addressing crowd sensing, web content, social behaviour recognition, etc.

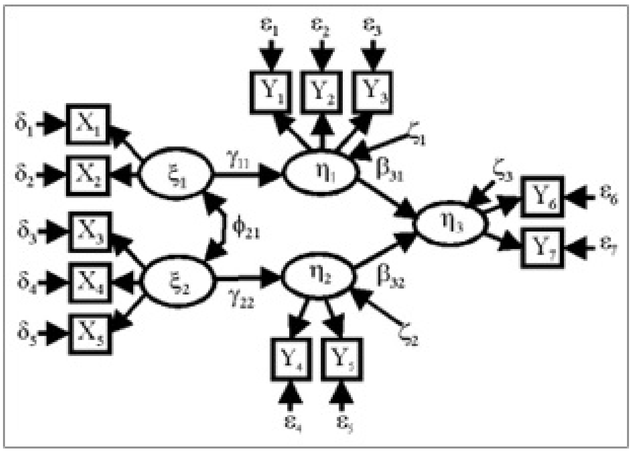


Figure 1: Abstract SEM diagram which could be applied for, e.g., workforce optimization. Ovals and squares indicate latent and observed variables, respectively. Arrows indicate the direction of impact.

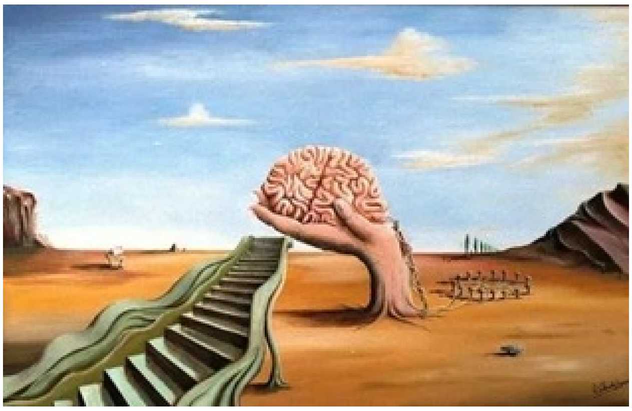
Multi-agent models

The first studies of social-behavioural phenomena concerned the modelling of large social networks using relational algebra and probability models that have given rise to mathematical sociology. Coleman, in his book "Introduction to Mathematical Sociology" (1964), showed how stochastic processes in social networks could be analyzed in such a way as to enable testing of the constructed model by comparison with the relevant data. For instance, agent-based socio-computational models can leverage social network analysis and cyber forensic methodologies, e.g. help in identifying deviant cyber bahaviors in social media platforms (Alassad et. al, 2019).

Game theory

The combination of multi-agent models and game theory has proved to be particularly fruitful. The theory of rational choice explains how an individual decides to participate or not in public affairs, as well as the logic of individual decision-making in collective affairs, but does not explain how members of the group communicate with each other to make the most advantageous decision for themselves and others. This is where game theory becomes relevant. Game theory encompasses a wide range of decision-making tasks (games), where the outcome of the choices of alternatives (decisions) is influenced by the actions of other individuals. Since people live in society, game theory is of particular practical importance (Bicchieri, 2003).

A classic example of game theory is the prisoner's dilemma. Robert Axelrod has developed a multiagent computer model in which agents interact with each other to solve this dilemma. He organized a competition of programs implementing various cooperation strategies to solve this dilemma and summed up the results of this competition, highlighting the features of the most successful strategies. The results are described in "The Evolution of Cooperation" (Axelrod, 1984). Axelrod discovered that if gaming had been repeated for a long time among many agents, each with different strategies, "greedy" strategies had eventually yielded bad results when more "altruistic" strategies were in place. He used this result to show a possible mechanism for the evolution of altruistic behaviour by natural selection from mechanisms that were originally purely egoistical.



Game theory found applications in helping develop ethical decision-making knowledge and skills (Jagger et al., 2015) as well as in modeling and simulation of impact and control in social networks (Agieva et al., 2019), modelling the behavior of economic agents as a response to information on tax audits (Kumacheva et al., 2019), and many other modelling tasks. An important category of game theory applications is group decision-making tasks in light of the optimal decision referring to the concepts of equality, sovereignty, anonymity, neutrality, including those used in politics. Arrow tried to create a way to correctly summarise personal preferences, i.e. to create a fair system of group decisions. But he only succeeded in formulating and proving the theorem that this was impossible. This theorem is sometimes called a theorem about the impossibility of democracy (Arrow, 1950). Arrow's theorem concludes that there is no way of harmonising the group decision satisfying all the basic concepts mentioned above.

Evidence-based solutions

Evidence-based decisions and policies can be the way to overcome paradoxes similar to those shown by Arrow's theorem. A clearly structured step-by-step process of evidence-based decision-making helps maximise the likelihood of the best (most advantageous) decision being taken. In order to make an informed decision, it is possible and necessary to build on (a) the results of the research, (b) the experience of the subject matter in question, and (c) the context in which the decision is taken (Banasiewicz, 2019). The evidence-based decision-making process consists of (1) identification of the need for a decision, (2) gathering of evidence, (3) identification of alternatives; (4) assessment of the evidence against the identified alternatives, (5) selection of the best alternative, (6) decision (action), and (7) evaluation of consequences. All parts of this process are important and cannot be neither omitted nor performed neglectfully.

Since people live in communities and their actions often directly or indirectly affect each other, decisions are often taken in groups. In this context, decision-making should contain "good" qualities, such as transparency, inclusiveness, openness, qualified leadership and a well-defined process. The achievement of all these qualities is an objective to achieve optimal results, but each decision-making situation is unique and the level to which these qualities can be achieved varies from situation to situation.

Multiple intelligence

Let us note that communication based on logical and verbal intelligence is a fundamental phenomenon of people's social behavior, although these intelligencies are only two of several types of intelligence characterising human beings. The theory of multiple intelligence was suggested by Howard Gardner, who in the 1980s described 7 types of intelligence (see Figure 2). Despite Gardner's proposed system of multiple intelligence, intelligence is generally perceived in society as logical/mathematical and verbal/linguistic competences, and standard IQ tests are designed specifically to investigate these abilities. According to Gardner, other forms of intelligence are undeserving of ignorance. Concerning this, there is a kind of discrimination against other ways of perceiving the world. Members of the society should understand this and be tolerant of each other. Tolerance means that all people should be able to develop their potential regardless of their type of intelligence. For this matter, the path is opened up through a creative society which understands that the best results can only be achieved by tolerating different types of intelligence.

On the other hand, given that the purpose of human knowledge is the truth, the priority given by society to logical and verbal intelligence is quite understandable. However, even without considering the issue of discrimination, restricting to these skills alone is flawed in itself. Research into computer science and mathematics in recent decades has revealed the limitations of logical

Figure 2: Constituents of the Gardner‘s multiple intelligence.

thinking, because logical-verbal intelligence can lead to unsolvable problems, to explore the reality which is, in fact, infinite, although we can use only finite number of symbols, etc.

Thus, it must be noted that society would benefit from accepting that intelligence is much more than the traditionally valued verbal and logical abilities. In order to ensure that the future society is united, creative and prosperous (Florida, 2002), and its members successfully reach the highest stage of the Maslow pyramid, i.e. self-realization, part of society's efforts should focus on developing the multiple intelligence of its members.

Conclusion

Thus, the discussed bridging between operationalisation and OR paves the way for the use of OR methods and tools in social sciences and humanities. In this way, it makes it possible to expand the field of OR and turn OR into a fundamental discipline similar to mathematics, physics or chemistry, since OR becomes not only science, but also philosophy or even ideology.

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Sequence building with narrow time windows in transport planning

by Marine Gautier (DORS Prize 2021 for best Master thesis within OR)

Satisfaction of time windows has recently become an increasing concern in transport planning. It is a critical issue for pick-up and delivery companies both with regards to customer satisfaction and competitiveness. In this master's thesis, linear programming and meta-heuristics were combined to develop high-quality solutions to an unusual travelling salesman problem combining both very narrow and very wide time windows. Beyond solving the initial problem, this master's thesis designed a general framework that could be used to solve various travelling salesman problems with different types of time windows.

The problem

The focus of this master's thesis was on a single-depot single-vehicle routing problem with time windows whose specificity was to combine both very narrow and very wide time windows. Besides, the departure time of the vehicle from the depot was unknown and to be identified which limited the use of basic recursions. For feasibility purposes, soft time windows were allowed but penalized. The task was to build a route for the vehicle such that all locations were visited and the time windows were respected as much as possible in order to minimize the distance travelled by the vehicle, the tour duration and the penalty costs associated to violations of the time windows.

Mathematical formulation

Let $G = (V, E)$ be a complete and directed graph where $V = \{0, ..., n\}$ is the vertex set and E the edge set. Vertices $i = 1, ..., n$ correspond to the customers whereas vertex 0 corresponds to the depot. An additional node $n + 1$ is considered and represents a copy of the depot.

Parameters

- d_{ij} associated to $(i, j) \in E$ represents the distance travelled by the vehicle between customer i and customer j (in km), with distances being asymmetric, $d_{ij} \neq d_{ji}$, $\forall (i, j) \in E$.
- t_{ij} associated to $(i, j) \in E$ represents the driving time of the vehicle between customer i and customer j (in min).
- s_i represents the time needed to service customer $i \in V$ (in min).
- $[e_i, l_i]$ describes the time window associated to customer $i \in V$, meaning the desired period of time to visit customer i (in min after midnight). The values e_0 and l_0 represent the earliest possible departure time and latest possible arrival time at the depot for the vehicle.
- δ^e and δ^l denote the fixed penalties associated to the violations of the time windows. δ^e defines the penalty for serving a customer earlier than the beginning of its time window while δ^l corresponds to the penalty resulting from a late service.

Variables

- $x_{ij} \in \{0, 1\}$, $\forall (i, j) \in E$, is a binary variable which indicates whether or not the vehicle is driving along the edge (i, j) in the solution.
- t_i, $\forall i \in V \backslash \{0\}$, is a continuous variable which denotes the time at which the vehicle serves customer i (in min after midnight). It is allowed that the vehicle arrives at a customer location and waits before serving the customer. Here, t_i corresponds to the actual time when the service of the customer begins and not the arrival time of the vehicle at the customer location.
- t_0 corresponds to the departure time of the vehicle from the depot and t_{n+1} corresponds to the arrival time of the vehicle at the depot. Thus, the tour duration is defined by $t_{n+1} - t_0$.
- α_i, $\forall i \in V \backslash \{0\}$, is a continuous variable which indicates, if applicable, how much time before the start of its time window customer i is served (in min).
- β_i, $\forall i \in V \backslash \{0\}$, is a continuous variable which indicates, if applicable, how much time after the end of its time window customer i is served (in min).

Objective

The objective function of the problem is composed of three distinct terms that can be ranked in decreasing order of level of criticality: (a) the minimization of the penalty costs induced by the violations of the time windows, (b) the minimization of the tour duration, and (c) the minimization of the distance travelled by the vehicle. Weights were defined to scale the different terms in the objective function according to their importance to a company and $\omega_1, \omega_2, \omega_3$ correspond to the weights associated to the different terms.

Some discussions led to conclude that the cost of driving 1km is somewhat similar to the cost it takes to drive 1 km ($\approx$ 1 min), thus it was reasonable to set $\omega_1 = 1unit/km$ and $\omega_2 = 1unit/min$.

Since early and late services of customers must be avoided as much as possible and soft time windows were allowed to guarantee there is a feasible solution to the problem, it was required to build a penalty function that would minimize the violations of the time windows. It was decided to use a quadratic penalty function which would lead to favor small violations for every customers instead of big ones concentrated around few customers. Besides, early services are more critical to a company, thus, they must be more importantly penalized than late ones. It was decided to arbitrary set $\delta^e = 10$ and $\delta^l = 1$ to assume it is worth to increasing the tour duration instead of realizing a late service by more than one minute to maximize the customer satisfaction. Consequently, $\omega_3 = 1unit/min^2$.

Constraints

The final mathematical model is the following :

$$
\begin{array}{lllr}
\text{minimize} & \omega_1 \cdot \sum_{(i,j)\in E} x_{ij} \cdot d_{ij} + \omega_2 \cdot (t_{n+1} - t_0) + \omega_3 \cdot \sum_{i\in V\setminus\{0\}} \delta^e \cdot \alpha_i^2 + \delta^l \cdot \beta_i^2 & & \\
\text{subject to} & \sum_{j\in V, j\neq i} x_{ij} = 1 & \forall i \in V, & (1) \\
& \sum_{j\in V, j\neq i} x_{ji} = 1 & \forall i \in V, & (2) \\
& t_i + s_i + d_{ij} \leq t_j + (1 - x_{ij}) \cdot M & \forall i \in V, j \in V\setminus\{0\}, & (3) \\
& t_j + s_j + d_{j0} \leq t_{n+1} + (1 - x_{j0}) \cdot M & \forall j \in V, & (4) \\
& t_i \leq l_i + \beta_i & \forall i \in V\setminus\{0\}, & (5) \\
& e_i - \alpha_i \leq t_i & \forall i \in V\setminus\{0\}, & (6) \\
& \alpha_i \leq \alpha_{max} & \forall i \in V\setminus\{0\}, & (7) \\
& \beta_i \leq \beta_{max} & \forall i \in V\setminus\{0\}, & (8) \\
& t_0 \geq e_0, & & (9) \\
& t_{n+1} \leq l_0, & & (10) \\
& t_i, \beta_i, \alpha_i \geq 0 & \forall i \in V\setminus\{0\}, & (11) \\
& t_0 \geq 0, & & (12) \\
& t_{n+1} \geq 0, & & (13) \\
& x_{ij} \in \{0,1\} & \forall (i,j) \in V^2 & (14)
\end{array}
$$

Constraints (1) and (2) ensure that all nodes are visited exactly once in the tour by ensuring they are entered and left exactly once. Constraints (3) and (4) ensure the sub-tours elimination and set the service times of customers. Constraints (5) and (6) allow to calculate, if existing, the violations of the customers time windows. Constraints (7) and (8) ensure that the violation of the time window is limited to some extent. These constraints were added after preliminary analyses which showed very small violations of the time windows in the solutions. Considering the benefits gained in terms of computational time when using flexible time windows instead of completely soft time windows, it was decided to reduce the solution space. This was even more acceptable to do so since very late and early services were undesired anyways. It was decided to continue the study with a model with flexible time windows constraints where $\alpha_{max} = 15min$ and $\beta_{max} = 180min$. Constraints (8) and (9) ensure the time window associated to the depot cannot be violated. Constraints (12) to (14) define the nature of the variables.

A feasible solution is a path starting at the depot ($i = 0$), ending at the depot ($i = n + 1$) and visiting all customers ($i = 1, ..., n$) as much as possible within their time windows. The route must hold within the depot time window.

The methodology

Several methods were investigated with the objective of building up the best possible solution to the problem in reasonable times. Due to the size of the problem, the main strategies to address the problem focused on the use of heuristics methods. Some construction heuristics were developed in order to create feasible solutions to the problem and two meta-heuristics, the *Greedy Randomized Adaptive Procedure* and the *Adaptive Large Neighborhood Search*, were used to improve the initial solutions found.

Construction heuristics

A two-stage methodology was used to design feasible solutions.

Stage 1 consisted of finding the optimal sequence to visit customers with narrow time windows only using the MIP formulation presented earlier and the optimization solver Gurobi (v.9.0). It provided an initial sequence S of customers to visit to be used as a basis for designing a complete feasible solution.

Stage 2 consisted of inserting the remaining customers at a minimal cost in the initial sequence S. Firstly, the well-known *cheapest insertion heuristics* was implemented. It was shown that finding for each customer and each possible insertion the insertion that minimized the objective function corresponded to solving a LP-model optimizing the service times for each customer in the tested sequence. Consequently, large running times were needed to build a feasible solution. To reduce the running time of building an initial solution, a second approach was tested and two additional construction heuristics were developed. It consisted of a first step identifying the nearest (or furthest) neighbor of the last customer inserted in the sequence S and a second step finding the best insertion for the selected customer. The second step also used the LP-model mentioned previously. This second approach helped to reduce the number of calls to the LP-model and decrease the running times of the construction heuristics without impacting the quality of the solutions obtained. Lastly, it was decided to implement an algorithm able to build a feasible solution to the problem without calling the LP-model. The framework of the *cheapest insertion heuristics* was used. However, for each customer and each possible insertion, instead of calling the LP-model to find the optimal service times for the customers in each tested sequence and then, the best possible objective value for the insertion, the service times as well as the departure and arrival time of the vehicle at the depot were manually calculated according to specific tailor-made rules. Two different rule systems were defined and two different construction heuristics were developed. Results showed that these construction heuristics performed very well and seemed to be a great compromise for building up high-quality solutions both in terms of objective value and running times.

Meta-heuristics

Greedy Randomized Adaptive Procedure

The *Greedy Randomized Adaptive Procedure* was used to investigate how non-ideal decision-making in a greedy algorithm might lead to access better solutions later on. Two greedy randomized constructors were developed based on the lastly described construction heuristics. The candidate list was defined as the list of the K best possible insertions at each stage of construction of the solution. The termination criteria was defined as a maximum amount of iterations to perform $Nmax$ to cope with the diversity of the instances.

Experiments demonstrated that the most performing algorithm based on the GRASP procedure was a hybrid version of the main algorithm using the two different greedy randomized constructors in combination within the termination criterion. The tuning of the algorithm was performed to determine each of its parameters. Several local searches were also tested but did not provide significant improvements of the results.

Algorithm 1 Tailor-made GRASP pseudo-code

```
Initialize parameter γ
S_best ← ∅
while not γ · TERMINATION CRITERIA do
    S_local ← GreedyRandomizedConstructionV1()
    if Cost(S_local) ≤ Cost(S_best) then
        S_best ← S_local
while not TERMINATION CRITERIA do
    S_local ← GreedyRandomizedConstructionV2()
    if Cost(S_local) ≤ Cost(S_best) then
        S_best ← S_local
```

Adaptive Large Neighborhood Search

The *Adaptive Large Neighborhood Search* was used since it was learned during the literature review that this recent meta-heuristics provided promising results for various routing problems. The reader can refer to [1] for a detailed description of the meta-heuristics.

Three destroy methods were used: the*k-random customers removal* which randomly selects k customers and remove them from the current solution, the *p-distance expensive customers removal* which aims to remove customers responsible for detours in the solution since they largely influence the objective function by increasing the distance travelled and consequently, most likely, also the tour duration. and the *q-most expensive customers removal* which aims to remove the most expensive customers with regards to the full objective function.

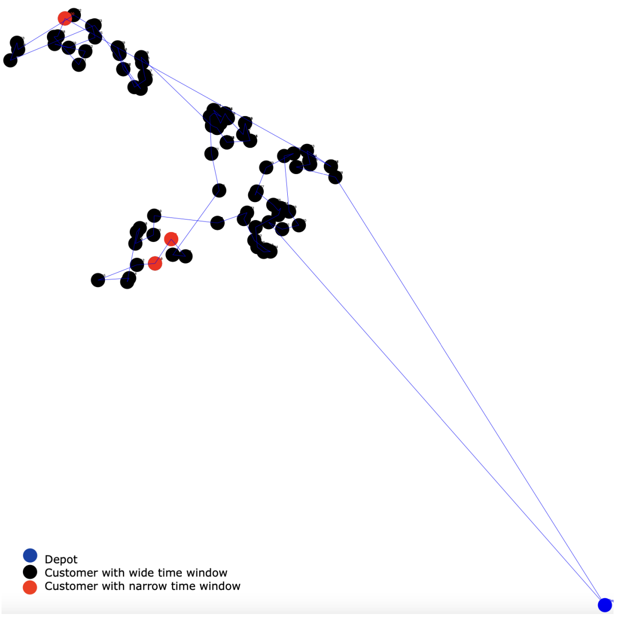
Four repairing methods were used, all based on the construction heuristics developed in the first part of the master's thesis : the combination of the nearest (or furthest) neighbor and the cheapest insertion heuristics as well as the two tailor-made construction heuristics.

Simulated annealing was used for the acceptance criterion and a classical weight procedure was used to for the destroying and repairing methods selection. The tuning of the algorithm was also performed to determine the ideal set of its parameters.

The results

The tailor-made construction heuristics demonstrated their ability to create high-quality solutions within seconds. The meta-heuristics helped to improve them at the expense of much longer running times. In 85% of the provided instances, the ALNS outperformed the GRASP. However, by analyzing in more details the solutions where the largest improvements were observed, it was noticed that the gains obtained using the ALNS algorithm corresponded to a reduction of the distance travelled by the vehicle by around 1 to 3 km and the reduction of the tour duration by less than 10 min. Then, it was concluded that for the provided set of instances the tailor-made construction heuristics were the best compromise between high-quality solutions and running times.

Nevertheless, these results were further investigated and it was discussed that, the limited performance of the improvement algorithms could partially be explained by the characteristics of the provided instances (e.g. depot located far from the customers, cluster of customers) whose initial solutions do not leave room for much improvement and limit the benefits of the developed algorithms. Additionally, it was underlined that the ALNS was not used to its fullest potential due to the use of two time-consuming repairing methods limiting the number of iterations performed in reasonable running times and consequently, the size of the solution space searched.



The methodology and especially the two-stage strategy used to design feasible solution was also discussed. This strategy aimed to tackle the most strategical customers first and it was assumed that it was better to avoid violations of the time windows and maximize the contribution to customer satisfaction since most of the time greedy procedures delay the insertions of difficult customers to insert. The results obtained with this strategy for the construction heuristics are satisfying, especially from the point of view of customer satisfaction since almost no violation of the time windows is observed in the solutions. the latter was entirely based on the construction heuristics. Then, every iteration of the algorithm starts with the basis for the solution obtained using the MIP-model. Consequently, in all solutions created within the algorithm, customers with narrow time windows are always visited in the same order no matter the number of iterations performed which restricts the solution space searched. It could be interesting to investigate a new GRASP algorithm using an alternative greedy randomized procedure, at least, to begin each iteration of the algorithm to analyze if that brings more diversity in the solutions created and enhances the search. This is not an issue in the ALNS algorithm since customers with narrow time windows can be selected as part of the destroy-and-repair process.

Extensive use

The characteristics of the instances were not considered when developing the tools so, they could be used as-is for many instances (e.g. where customers are not gathered within clusters). Besides, by small changes to the MIP-model constraints used, they could also solve other routing problems combining both narrow and wide time windows (e.g. hard and/or soft time windows).

However, there are some obvious limitations to the extensive use of the tools mostly related to the use of linear programming. Using the MIP-model in the construction heuristics necessarily implies longer running times for building up solutions when the size of the sub-problem increases. Thus, the current tools are not expected to scale well to solve larger instances or instances with a larger absolute amount of customers with narrow time windows. Besides, the sizes of the time windows of the customers in the instances also influence the running times required to solve the MIP-model and the larger the narrow time windows, the longer the running times. A first suggestion to overcome these limitations could be to find an alternative way to build the basis for the sequence instead of using the MIP-model for the sub-problem at the expense of the quality of the solutions created (e.g. using a heuristics which selects the customers in an increasing order of opening of their time windows). An other alternative is a new definition of the criterion to identify the customers to include in the sub-problem (e.g. select the K customers with the narrowest time windows for the sub-problem). Both also have the advantages to keep a reasonable size of the sub-problem and keep targeting the most strategical customers to the business.

All things considered, this thesis provides a good general framework that could solve many travelling salesman problems combining both wide and narrow time windows.

References

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Underground mine scheduling by combining logic-based Benders decomposition and heuristics

by Emil Lindh and Kim Olsson (2021 best Master thesis within OR in Sweden)

The excavation of an underground mine is a complex operation that requires careful planning and is usually done in several phases with different time horizons. The two components of the planning process that we address in this paper are the so-called extraction plans and the short-term scheduling. The extraction plans describe when a certain amount of ore from a certain part of the ore body should be excavated. The short-term scheduling then executes these plans by scheduling the activities and the machines involved in the excavation process. The short-term scheduling is many times done manually, which is a time consuming task prone to errors. At the same time, it is a vital stage in the planning process. Thus, a way of optimizing and automating the short-term scheduling could yield great improvements.

The goal of our master thesis was to use a logic-based Benders decomposition [2] approach for solving a short-term scheduling problem for a cut-and-fill mine of the mining company Boliden. The master thesis project was conducted in collaboration with Boliden who provided realistic data and domain knowledge.

Underground mining and difficulties

The extraction of minerals can be done in many ways. Which method that is appropriate is dependent on the characteristics of the ore body. For example, when the ore body is close to the surface it is sometimes appropriate to use the open-pit method. In cases where the ore body is horizontal, the room-pillar method method is a common method. In the case of the ore body being more vertical, a common method is the cut-and-fill method, which is the method of the mine subjected to our problem.

In the cut-and-fill method, the ore is excavated in horizontal segments. Once the ore in one segment is excavated, the excavated void is filled with concrete, sand and waste rock. The filling is then used as a platform when excavating the next segment above. Usually, excavation occurs at many places at once. The tunnels created when excavating the rock will be referred to as drifts, and the end of each drift will be called the face. The excavation of the ore in the cut-and-fill method is done through a number of stages that together form an excavation cycle. In each cycle, a block of ore is removed from the ore body. The exact activities in the cycle may vary from mine to mine.

The cycle begins with drilling holes in the rock mass of the face, followed by charging explosives and detonating them to loosen the rock. The detonation produces toxic blast fumes which need to be ventilated before the next activity can begin. The ore that has been separated from the face is then loaded away. Scaling and cleaning is performed to remove loose rock to ensure the safety of the drift. When the loose rock has been removed, the roof and walls are sprayed with concrete and bolted to further secure the area. The complete cycle is illustrated in Figure 1.

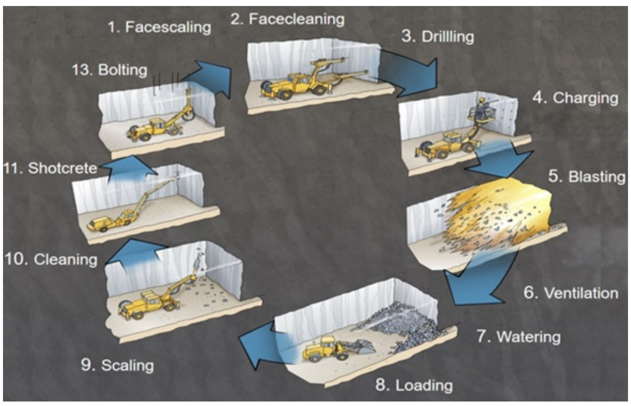

Figure 1: All activities included in a cycle. Image courtesy of Boliden AB.

As mentioned in the introduction, the short-term scheduling is the realization of an underlying extraction plan which describes how many blocks of ore that are to be excavated at each face at a certain point in time. In reality, this means there is always a priority order between all the faces depending on the nature of the extraction plans as well as how well the previous short-term plans have been executed.

A common goal of the scheduling is to minimize the sum of the end times of the last tasks at each face, thus maximizing production in some sense. In discussions with Boliden, we decided to relax this goal to instead minimize the total number of used shifts. This enabled the heuristic approach for solving the problem that we will present.

To conclude, the short-term scheduling problem considered in our master thesis is to schedule a certain number of excavation cycles at a certain number of faces, using as few working shifts as possible, while respecting the priority order between the different faces.

Algorithm

Previous attempts at solving similar problems have been made, most often modeling it as a k-stage hybrid flow shop and using constraint programming purely or iteratively to either solve or at least find feasible solutions. However, to solve this scheduling problem for realistic problem instances, the pure solution methods fall short [1]. To be able to tackle realistic problem instances, we proposed a priority-based rolling horizon heuristic. In the proposed method, we utilize both the priority order between the faces induced by the extraction plans, as well as our relaxation of the objective function.

The main idea behind the method we propose is to divide the schedule into batches of tasks, and to iteratively solve a larger and larger part of the entire scheduling problem by adding another batch to the problem each iteration. When a batch is scheduled, some of the scheduling decisions - which shift to schedule the tasks in, and which machines that should execute the tasks - are fixed, while the exact start and end times of all the tasks are redecided upon each iteration. This way, the solution time scales better with the size of the problems.

To schedule each batch, we propose a logic-based benders scheme [2], which divides the problem into two separate problems - a master problem and a sub problem - and resolves both of these two problems iteratively until a solution has been found. The master problem assigns machines to tasks and tasks to working shifts (also referred to as time windows) while using as few working shifts as possible. The sub problem schedules the tasks such that the master problem decisions are respected. If there is a feasible schedule, the problem is solved. But if the master problem decisions yield an infeasible solution, the master problem is resolved with a set of constraints added to it. The added constraints are called cuts, whose job is to remove solutions that are proven infeasible from the master problem. The solution process of the logic-based Benders scheme is illustrated in Figure 2.

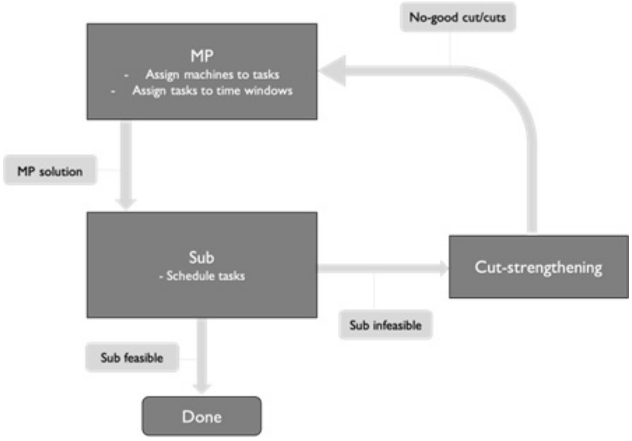

Figure 2: Illustration of the Logic Based Benders decomposition.

Results

To evaluate our proposed method, we compared it against a pure constraint programming model for a couple of different problem instances with randomized priority orders as well as a constraint programming model with the aforementioned constructive heuristic. All of the instances consisted of around 240 tasks. Since the constraint programming method was not able to provide a proof for an optimal solution in a feasible time, it was given the same amount of time as our proposed method needed to solve the problem.

The results show that our proposed method outperforms the others both in solution time and quality of schedule in all instances and priorities except one, and seems especially efficient when the ratio of cycles to faces is high. Our hope is that our work will be useful for future research in Logic Based Benders decomposition applications.

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Stockholm Optimization Days 2022

by Jan Kronqvist, Anders Forsgren och Per Enqvist (Organisationskommittéen)

Den 16:e till 17:e juni organiserade vi på avdelningen för Optimeringslära och systemteori på KTH Stockholm Optimization days. För de som har varit med en längre tid kan det här evenemanget vara bekant. Det organiserades första gången 1990 och var en årlig småskalig konferens som lockade både internationella och lokala forskare och hade en social och avslappnad atmosfär. Genom denna etablerades flera långvariga kontaktnätverk. Den har dock sedan år 2000 legat vilande. Efter pandemin och dess isolering kände vi att det fanns ett behov för optimeringsintresserade att träffas igen och det verkade vara ett utmärkt tillfälle att återstarta Stockholm Optimization days.

Konferensen höll till på KTHs campus i byggnaden kring borggården där vi hade lokaler för både seminarier, postersessionen, fika och diskussioner.

En av idéerna med konferensen var att välkomna en bredare grupp inom optimering, som skulle inkludera både akademin och industrin, med utvecklare av både programvara såväl som teori och metoder. Ett tema för årets konferens var blandad heltalsoptimering.

Vi lyckades få nio plenarföredrag från toppen av vår önskelista. De inbjudna talarna var Ambros Geixner (Zuse Institute Berlin, Germany), Ann-Brith Strömberg (Chalmers Tekniska Högskola), Claudia D´Ambrosio (CNRS & Ecole Polytechnique, Frankrike), Elina Rönnberg (Linköpings Universitet), Joey Huchette (Google Research, USA), Pontus Giselsson (Lunds Universitet), Robert Weismantel (ETH Zürich, Schweiz), Stephen Boyd (Stanford University, USA) och Thiago Serra (Bucknell University, USA).

Förutom plenarföredragen så hade vi två parallella sessioner med 16 presentationer. Eftersom det var fler inskickade bidrag än vi kunde ta emot så fick vi prioritera presentationerna från de som kom utifrån. Vi kunde därmed även erbjuda 18 poster presentationer av väldigt hög kvalitet som bidrog till många intressanta diskussioner. De inbjudna talarna utformade ett kommittee som valde ut fem posters till ett bästa posterpris, nämligen Sara Frimodig, Ivar Bengtsson, Ida Åkerholm, Björn Morén och Xiaoyu Wang.

Dessutom så hade vi en industrisession där några av våra sponsorer beskrev hur de använde optimering och vad de såg för behov och metoder som de prioriterade. De som sponsrade var Digital futures, GAMS, Gurobi Optimization, H&M, MOSEK, RaySearch Laboratories, Volue och Brummer & Partners MathDataLab.

De som deltog i konferensbanketten fick uppleva en busstur genom ett ovanligt kraftigt sommarregn, som slutade lagom till att vi kom fram till Ulriksdals värdshus. Den goda stämningen som infunnit sig under dagen förstärktes under middagen och diskussionerna gick långa. Det samlade intrycket var att alla uppskattade konferensen och allt runt omkring. Chansen att höra om de senaste utvecklingarna inom området, knyta nya kontakter eller återknyta till gamla och att bara få chansen att träffas och utbyta erfarenheter med andra inom området med gemensamma problem eller nya lösningar. Detta inspirerar oss att fortsätta driva konferensen framöver.

Jan Kronqvist is an Assistant Professor in Optimization and Systems Theory at the Department of Mathematics at KTH Royal Institute of Technology in Sweden. His research focuses on mixed-integer optimization, specifically on algorithm development, strong convex relaxations, and mixed-integer optimization for AI/ML.

Operations and Supply Chain Day 2022 in Aarhus

by Julia Pahl and Hartanto Wong

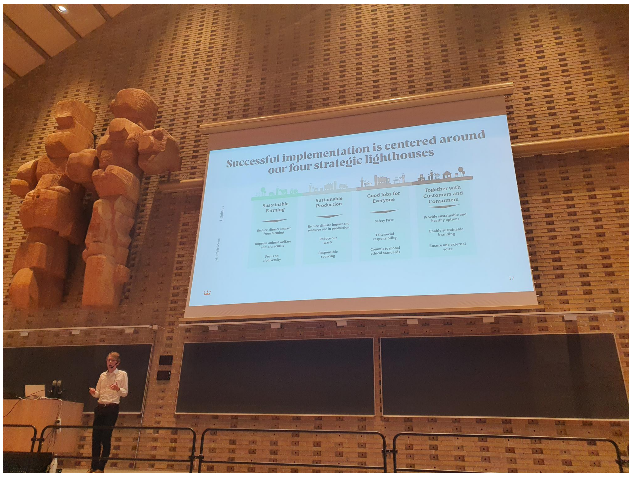
On the 6th of October, the "Operations and Supply Chain Day 2022" organized by the Cluster for Operations Research, Analytics, and Logistics (CORAL) at Aarhus University in collaboration with the Danish Operations Research Society (DORS) took place.

It was a lovely sunny autumn day and, despite the good weather outside, many students, practitioners, and professors attended the program which started with breakfast and the first opportunity for networking, chatting, and meeting colleagues and friends.

The program's central theme on sustainability was well selected by the organizers Ata Jalili Marand, Sune Lauth Gadegaard, Christian Larsen, and Hartanto Wijaya Wong. The program was comprised of four presentations given by practitioners from Implement Consulting Group, Danish Crowns, Novo Nordisk, and Rema 1000.

Mike Weisbjerg from Implement Consulting Group was the first speaker to introduce the audience to Sustainability in Supply Chain Planning and the support for companies to become more sustainable through planning. The supply chain design is vital from a sustainability perspective, but the planning aspect is just as important to enable a more sustainable decision making in operations.

The second talk was from Jakob Blaavand from Danish Crown with the topic of Data and Analytics for a Sustainable Future for Food. It was very interesting to note how Danish crown is making a great effort in collecting and analyzing data to become more sustainable. An important part in the process chain is constituted by the farmers which is also shown by the life cycle analysis of meat production. As a result, effort needs to be put into the collaboration with the farmers.



After a nice lunch time, the third talk was given by Eike Raft Vinther and Stephane Chong from Novo Nordisk on the Sustainable Supply Chain of Novo Nordisk that emphasis on getting things done now together with their stakeholders to make the supply chain become more sustainable. This talk shows that decarbonizing supply chains requires a multi-disciplinary effort to optimize operations and investments in new technologies. The talk also touched upon the importance of social aspect of sustainability exemplified by the project providing affordable care for vulnerable patients in Africa.

Finally, Kristina Hove Østergaard presented the Sustainable Efforts in Retail Logistics at Rema 1000. The talk gave inspirations that a lot can be done to enhance sustainability in the retail logistics. Some examples include: Dividing trailers into two temperature zones for shipping frozen and refrigerated goods; and Using CO2-cooled trailers for shipping frozen and refrigerated goods to replace diesel motors.

Julia Pahl is an Associate Professor at the SDU Section for Engineering Operations Management, Department of Technology and Innovation, University of Southern Denmark with special interest into operations research for supply chain management in various industrial areas such as manufacturing and maritime shipping.

Hartanto Wong is Associate Professor at CORAL, Department of Economics and Business Economics, Aarhus BSS, Aarhus University. His research interests include the broad areas of supply chain network design, service parts logistics and product line design problem, with OR/analytics as main methodology.